# Defeasible rule-based arguments with a logico-probabilistic foundation

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university of groningen

How do

**argumentation**, **logic** and **probability**

relate to each other?

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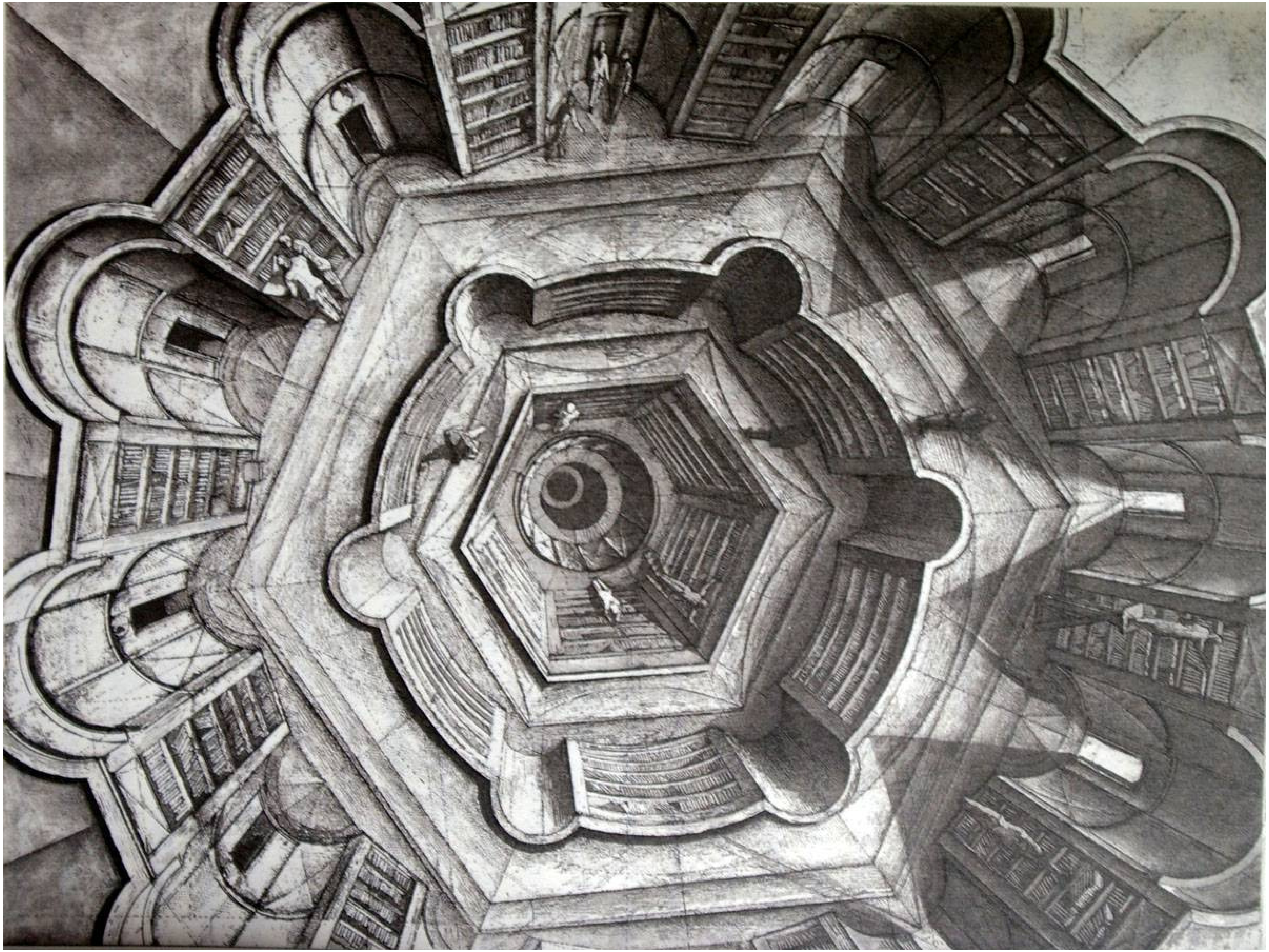
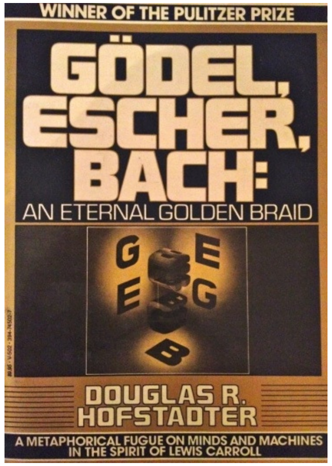
***Answering this question***
***is not the aim of this exercise.***

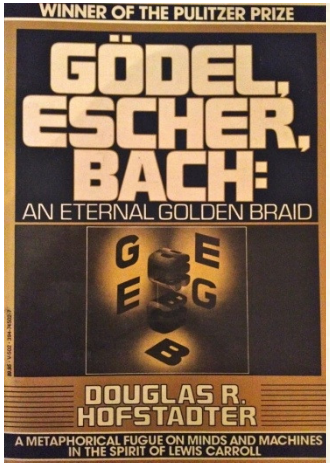


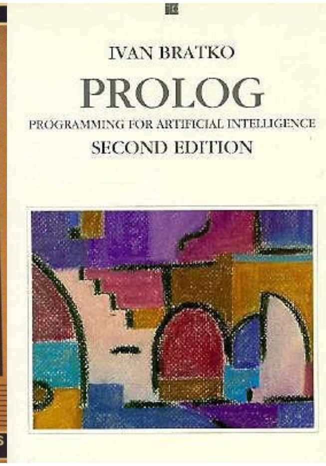
How does **intelligent agency** work?

What is the role of **argumentation**?




1. AI is possible --- and mathematical logic is relevant.



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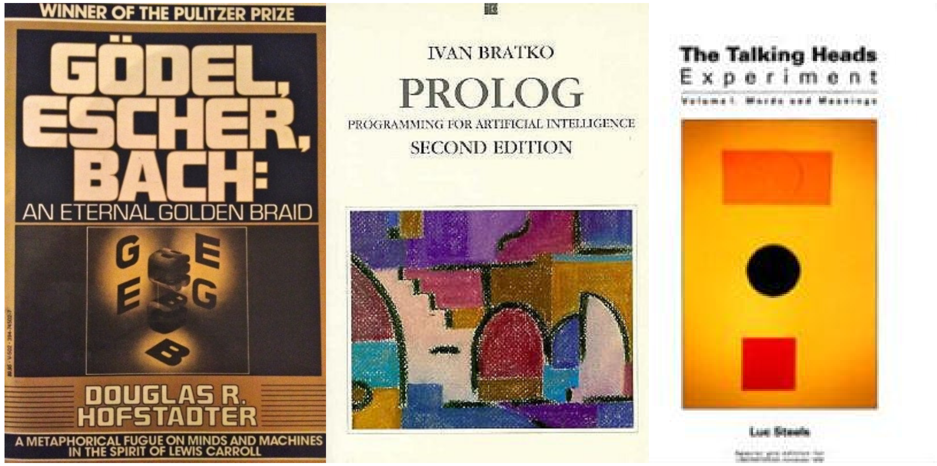
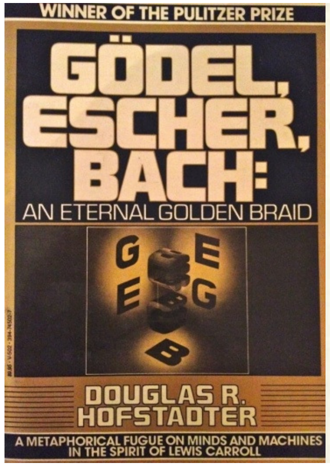
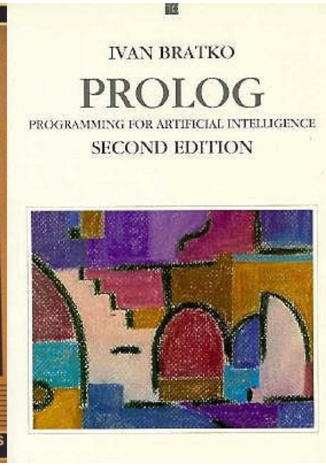
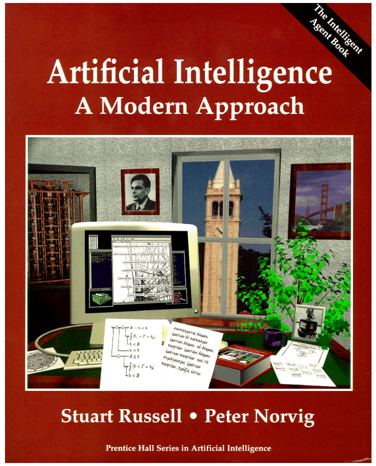



1. AI is possible --- and mathematical logic is relevant.
2. For AI, computional logic is the way to go.
3. AI is all about interaction with the environment and communication.





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2. For AI, computional logic is the way to go.
3. AI is all about interaction with the environment and communication.
4. AI is about autonomous agents that learn.

The view on knowledge representation and reasoning shifted roughly in parallel:

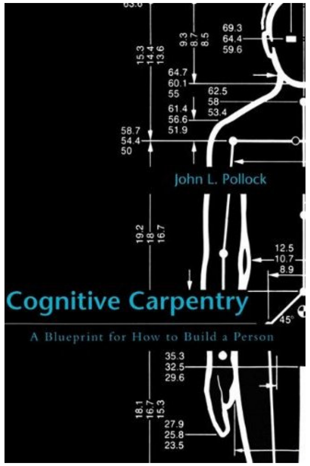
1. Classical logic
2. Nonmonotonic logic
3. No logic
4. Probability theory

logic-based AI *versus* probability-based AI

***Meanwhile …***

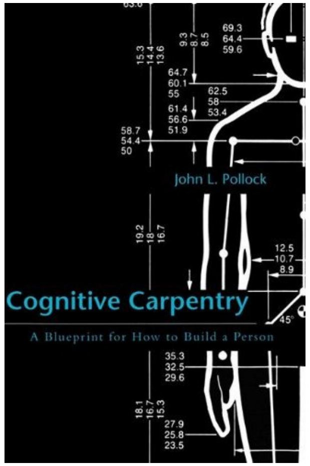
# Pollock on argumentation & AI (1995, 2010)

1. There are **kinds of defeaters**: undercutters and rebutters.
2. Argument **structure determines warrant**.
3. It is relevant to **classify defeasible reasons** (deductive reasons, perception, memory, statistical syllogism, induction).
4. A **computational perspective** is relevant (cognitive architecture OSCAR).



# Pollock on argumentation & AI (1995, 2010)

1. Arguments can have **different strengths**, and conclusions can differ in their degree of justification.
2. **Sufficiently strong** arguments provide a defeasible reason for the conclusion.
3. Degrees of justification **do not work like probabilities**.
4. Degrees of justification should be **computable**, and a probabilistic account precludes that.



logic-based AI *versus* probability-based AI

*Argumentation*

AI without a dichotomy

Logic-based and probability-based approaches can be integrated in a way that makes sense, by using an argumentation perspective.

These are **adolescents**.

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Period: **1970s, 1980s**, something like that.

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Is this **America**? Surely **the West**.

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These must be **smart priviliged kids**.

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O is it about the black boy (can you say black these days?). No idea who that is. It's **not Michael Jackson**.

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**Obama**!

**Jump to conclusions**

in order to

**make sense**

of the

**partial information**

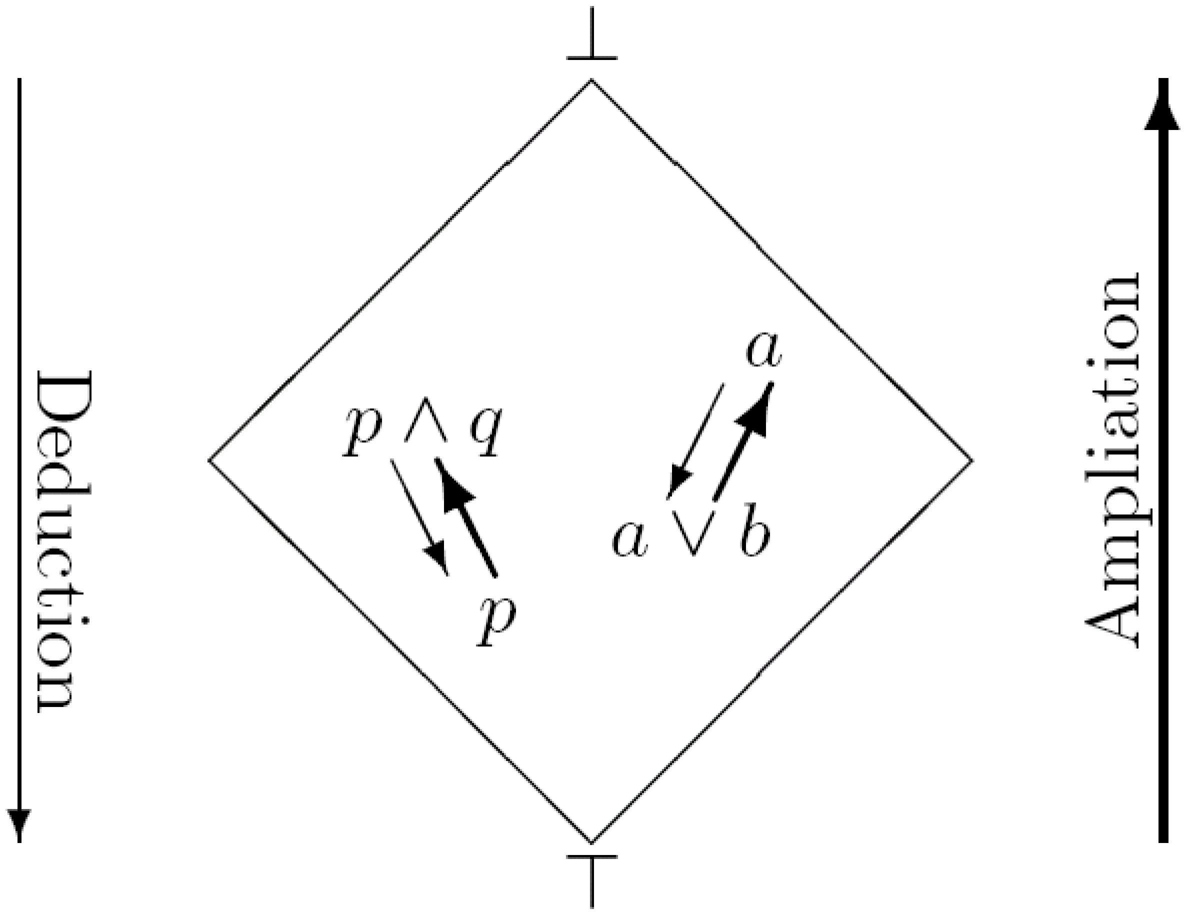
available

using **knowledge and evidence**

# Some ingredients

1. Ampliative arguments model jumping to conclusions.
2. Arguments are valid when they do not jump too far.
3. Jumping farther decreases (does not increase) argument strength.
4. Jumping to conclusions is defeasible.

# Ampliation & deduction



**Expats**

↑

**1970s, 1980s**

↑

**Adolescents**

↑

**Picture**

# Useful notion: *The case made*

**Picture, Adolescents, 1970s, 1980s, Expats**

↑

**Picture, Adolescents, 1970s, 1980s**

↑

**Picture, Adolescents**

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**Picture**

Jumping from $\varphi$ to $\psi$
*corresponds to*
Jumping from $\varphi$ to $\varphi \wedge \psi$

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Picture, Adolescents, 1970s, 1980s, Expats

↑

Picture, Adolescents, 1970s, 1980s

↑

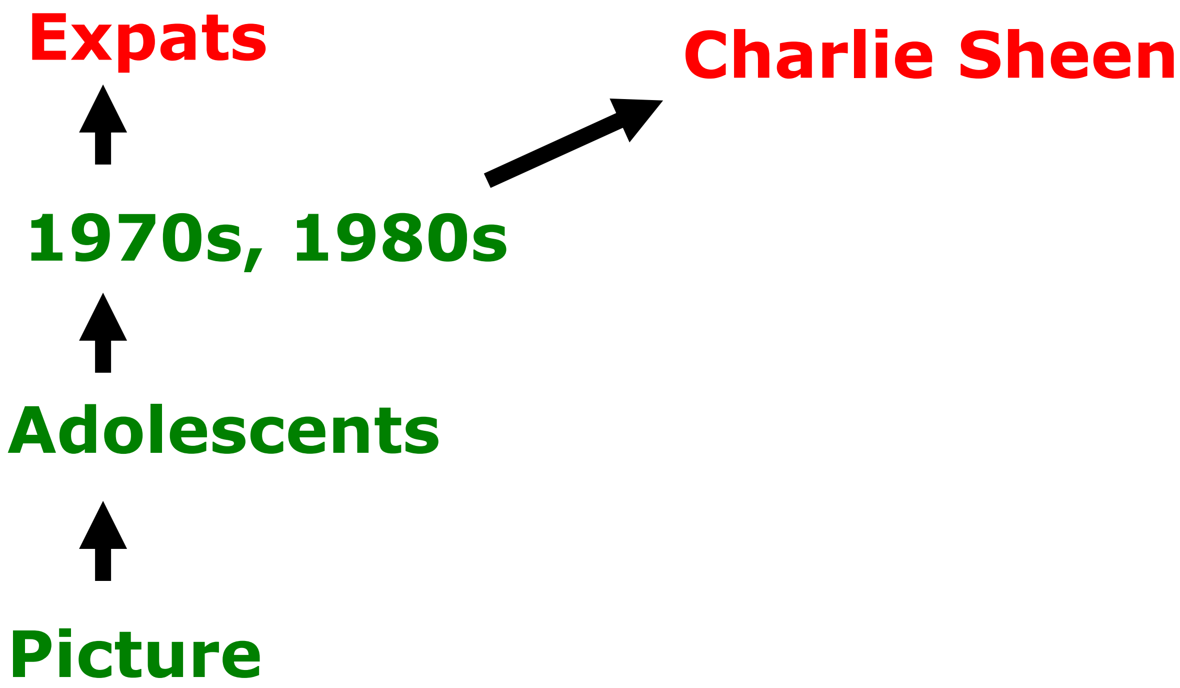
Picture, Adolescents

↑

Picture

**Global coherence (cf. the role of narrative)**

Jumping from $\varphi$ to $\psi$
*becomes*
Jumping from $\varphi$ to $\varphi \wedge \psi$


Expats
Charlie Sheen
1970s, 1980s
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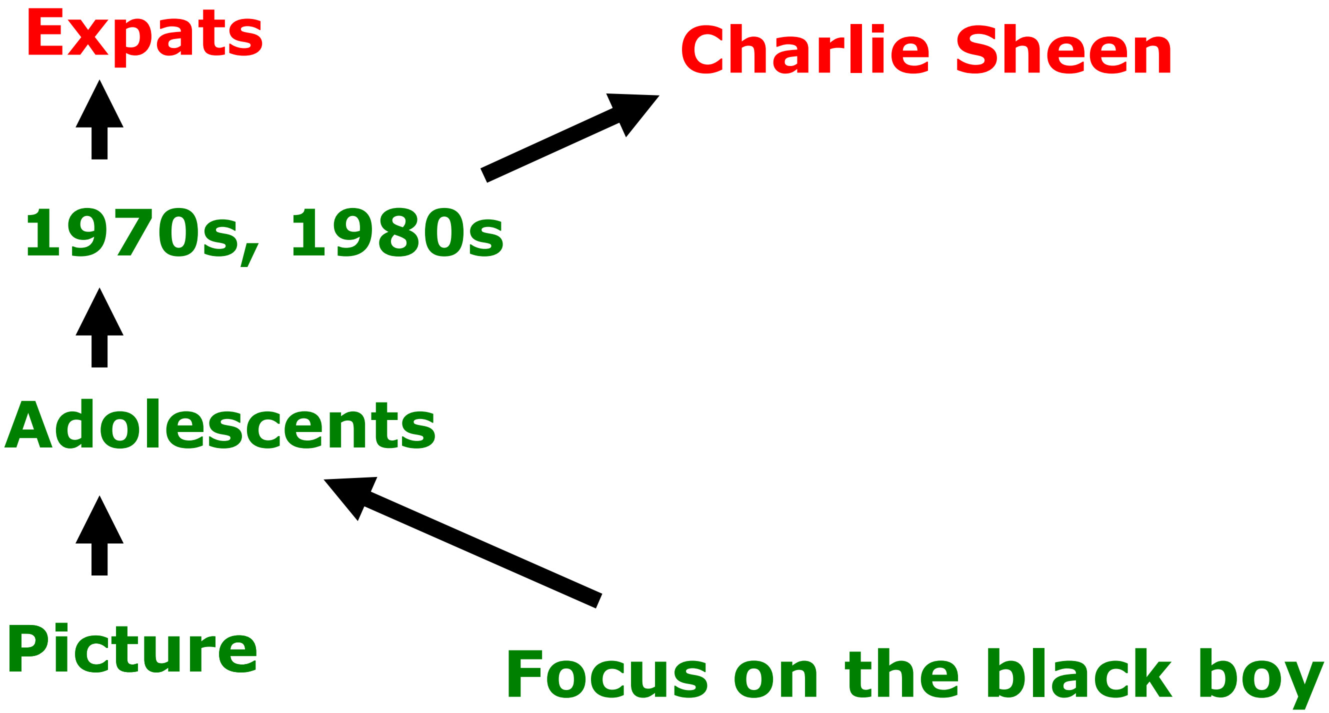
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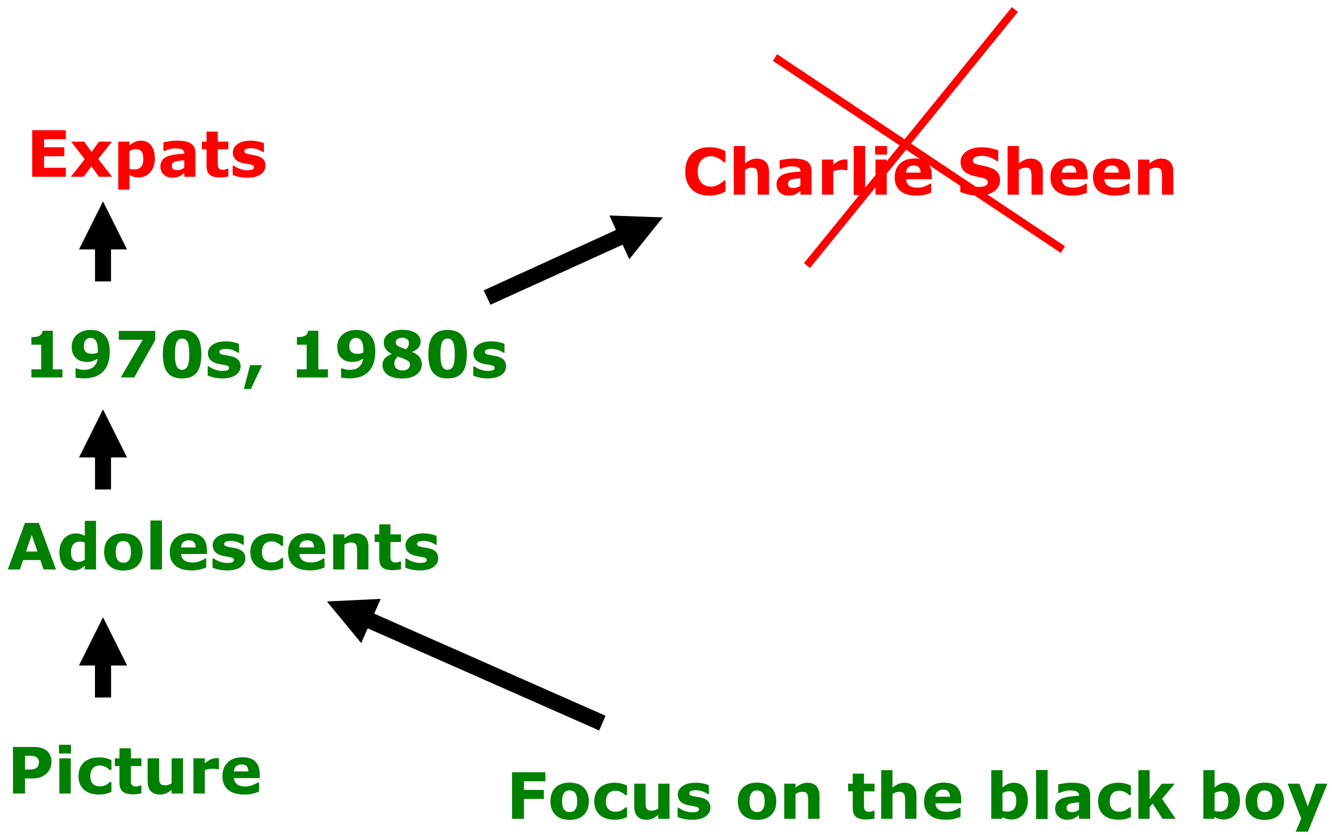

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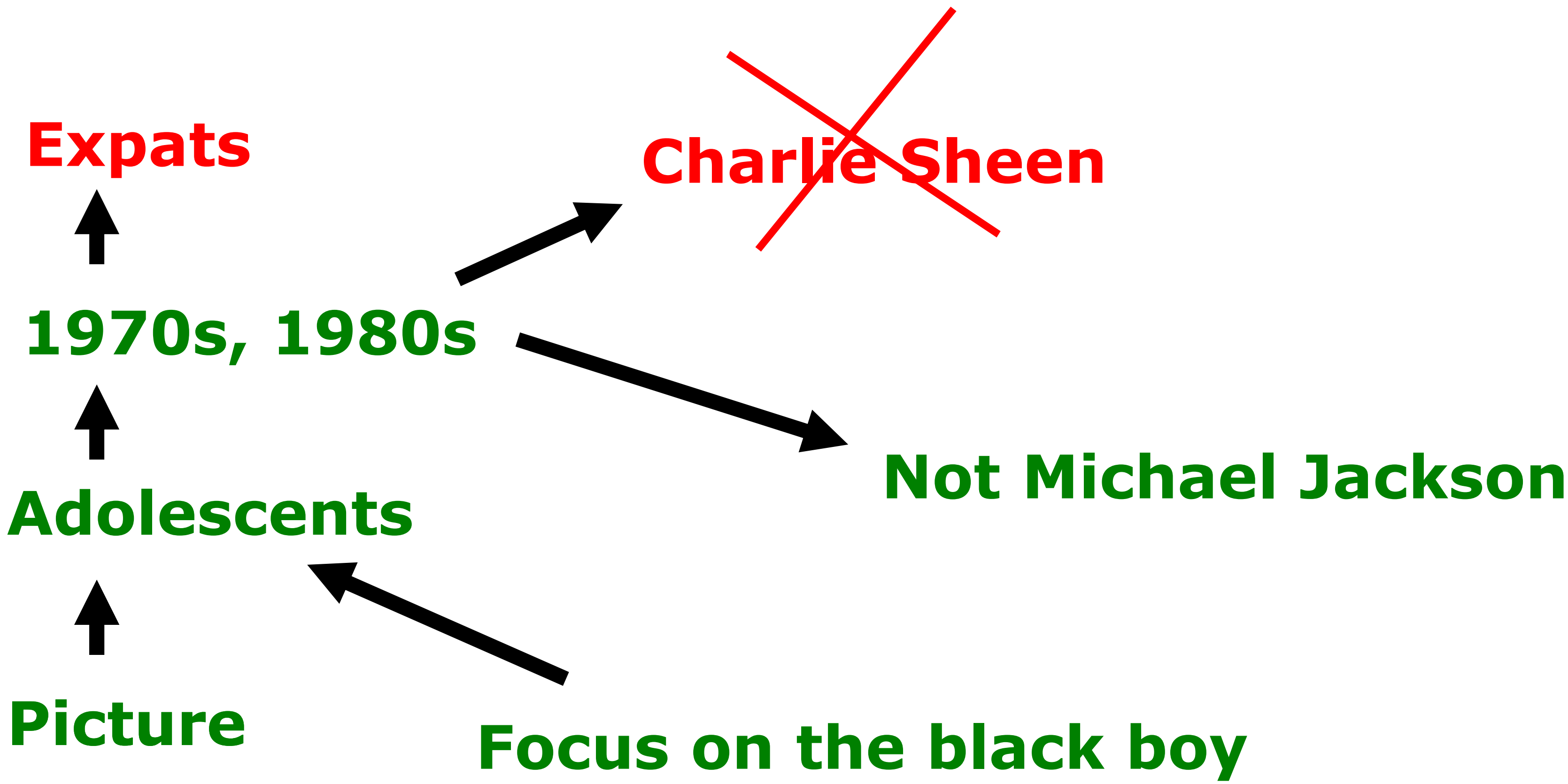
Expats
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1970s, 1980s
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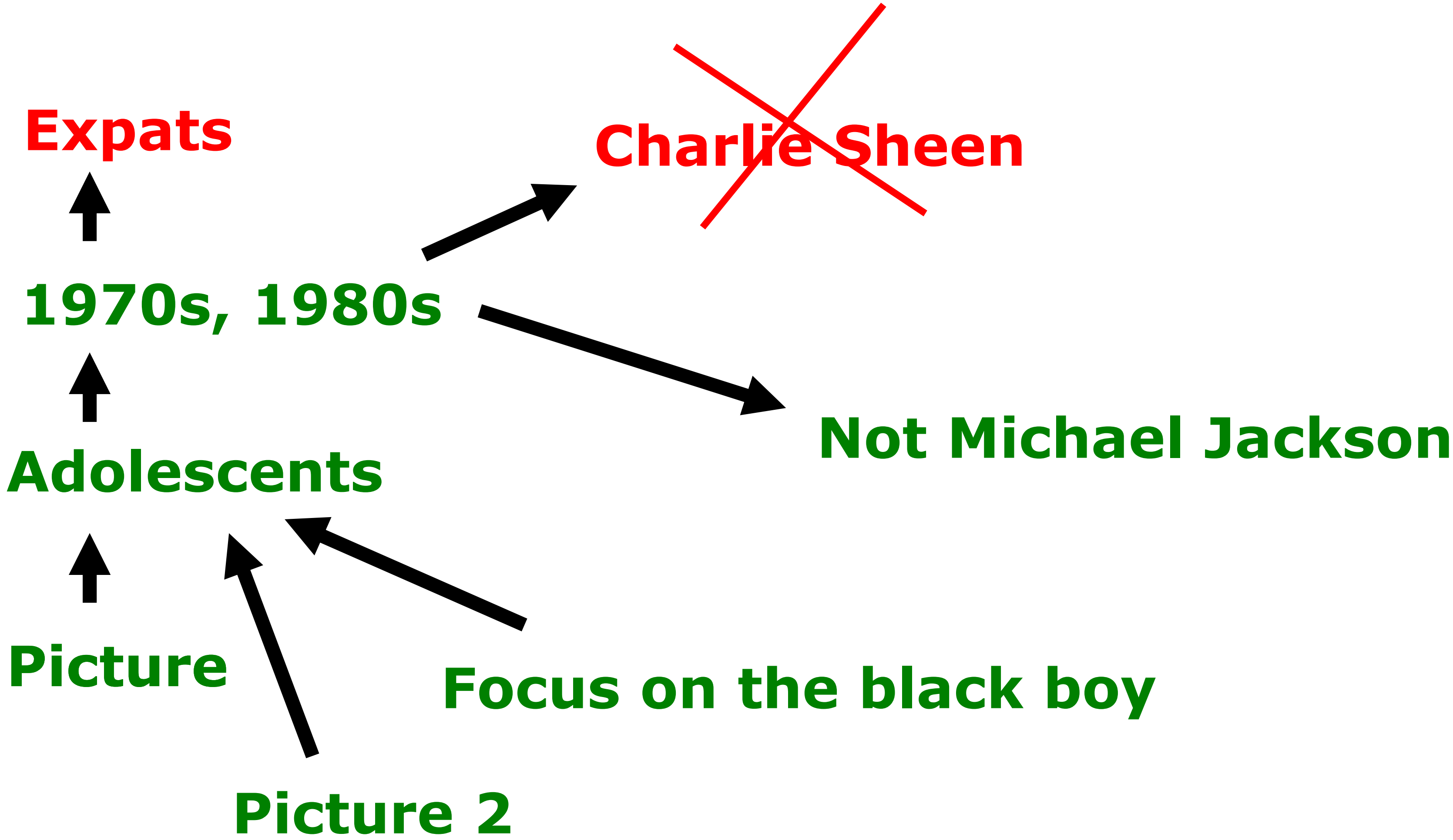
Expats
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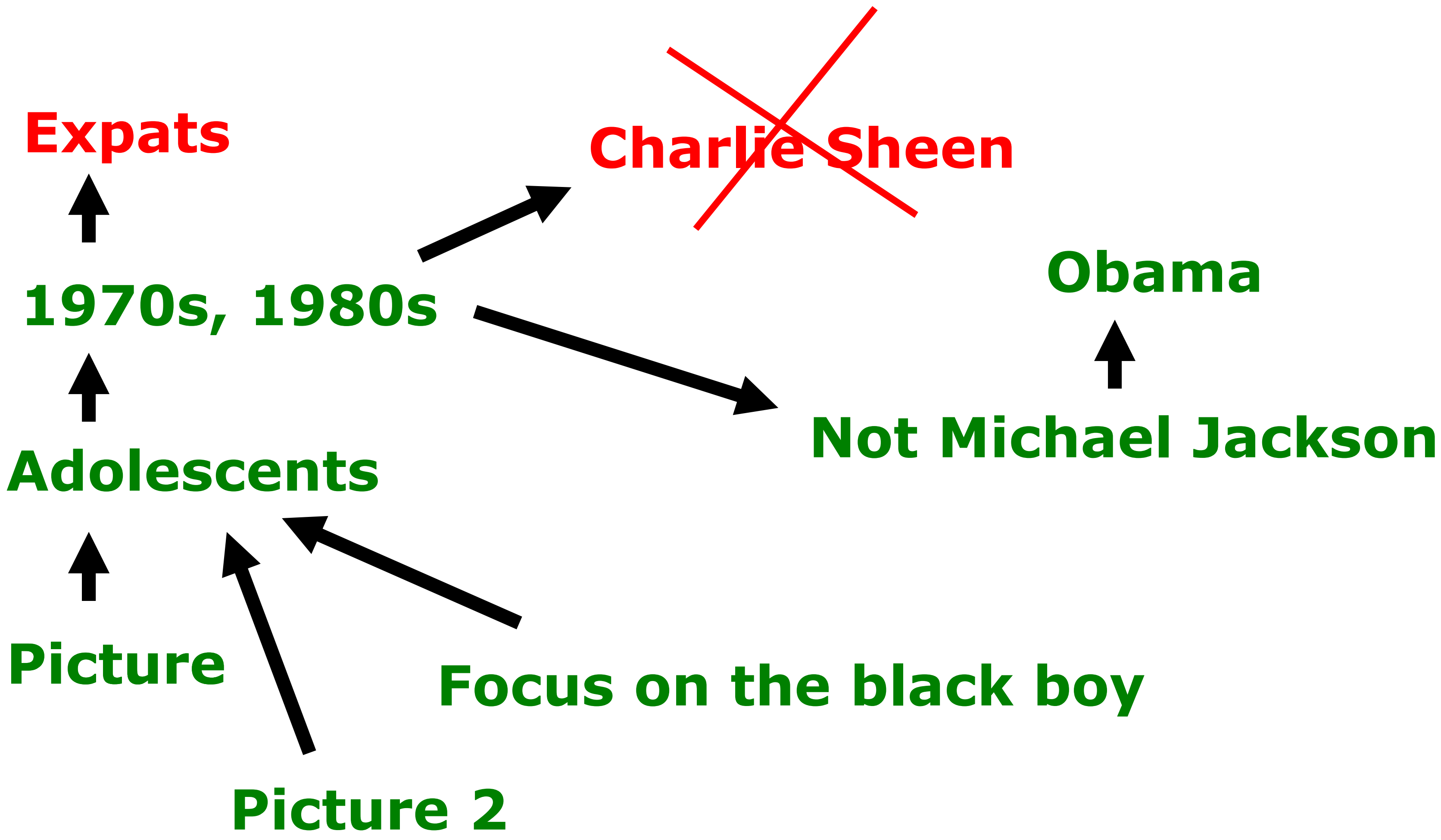
Adolescents
Picture
Picture 2
Focus on the black boy


Expats
Charlie Sheen
1970s, 1980s
Obama
Adolescents
Not Michael Jackson
Picture
Focus on the black boy
Picture 2

# Properties: nonmonotonic inference

| | |
|---|---|
| (LE) | If $\phi \mid\!\sim \psi$, $\vdash \phi \leftrightarrow \phi'$ and $\vdash \psi \leftrightarrow \psi'$, then $\phi' \mid\!\sim \psi'$. |
| (Ant) | If $\phi \mid\!\sim \psi$, then $\phi \mid\!\sim \phi \wedge \psi$. |
| (PR) | If $\phi \mid\!\sim \phi \wedge \psi$, then $\phi \mid\!\sim \psi$. |
| (R) | $\phi \mid\!\sim \phi$. |
| (RW) | If $\phi \mid\!\sim \psi \wedge \chi$, then $\phi \mid\!\sim \psi$. |
| (CCM) | If $\phi \mid\!\sim \psi \wedge \chi$, then $\phi \wedge \psi \mid\!\sim \chi$. |
| (CCT) | If $\phi \mid\!\sim \psi$ and $\phi \wedge \psi \mid\!\sim \chi$, then $\phi \mid\!\sim \psi \wedge \chi$. |

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**Conjunctive Cautious Monotony**

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**Conjunctive Cumulative Transitivity**

The (And) and (Or) properties are not assumed.

(And) would block the possibility of **distinct** reasonable jumps.

(Or) would imply that **settling a choice** cannot give new consequences.

| *A magnitude* | | *An order of magnitude* |
| --- | --- | --- |
| 0.30 | **Picture, Adolescents, 1970s, 1980s, Expats** | 1 |
| | ↑ | |
| 0.90 | **Picture, Adolescents, 1970s, 1980s** | 2 |
| | ↑ | |
| 0.95 | **Picture, Adolescents** | 2 |
| | ↑ | |
| 1.0 | **Picture** | 2 |

1. *If* $\vdash \varphi \leftrightarrow \psi$, *then* $O(\varphi) = O(\psi)$.
2. $O(\bot) \leq O(\varphi) \leq O(\top)$.
3. $O(\varphi) \geq max(O(\varphi \wedge \psi), O(\varphi \wedge \neg\psi))$.
4. *If* $\psi \vdash \varphi$, *then* $O(\varphi) \geq O(\psi)$.
5. $\varphi \mid\!\sim \bot$ *if and only if* $O(\varphi) = 0$.
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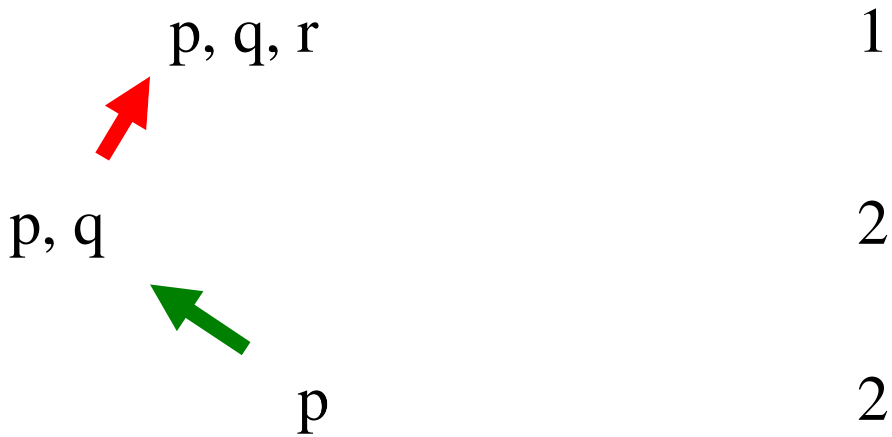
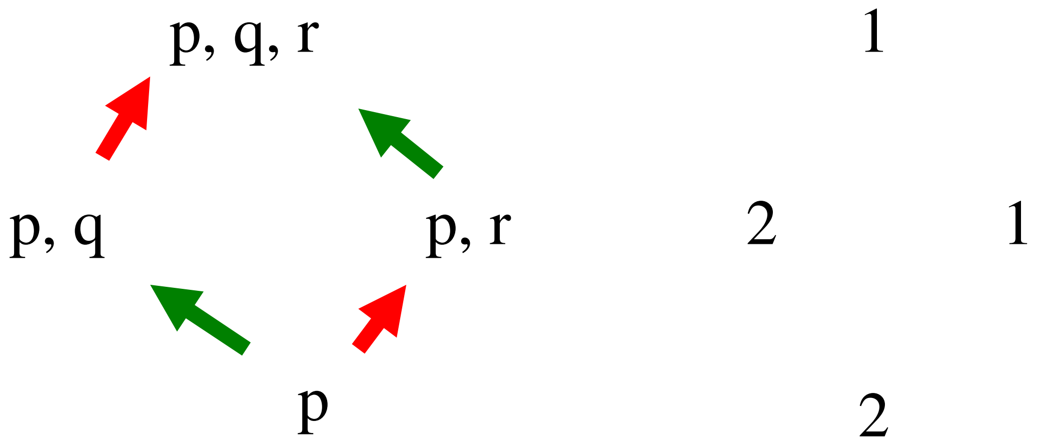
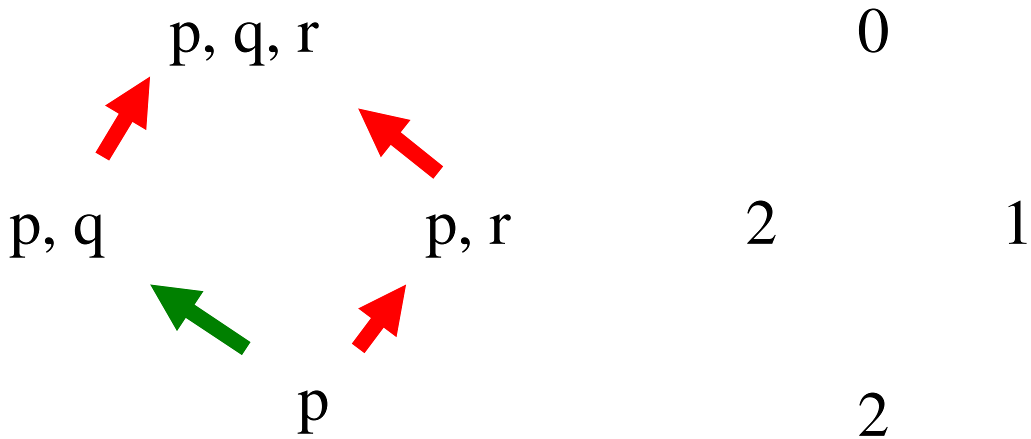

p, q, r
1
p, q
2
p
2




The rule ‘If p, then q’ is also applicable in the context ‘r’.




The rule ‘If p, then q’ is not applicable in the context ‘r’.

The rule ‘If p, then q1’ has exception ‘r’.

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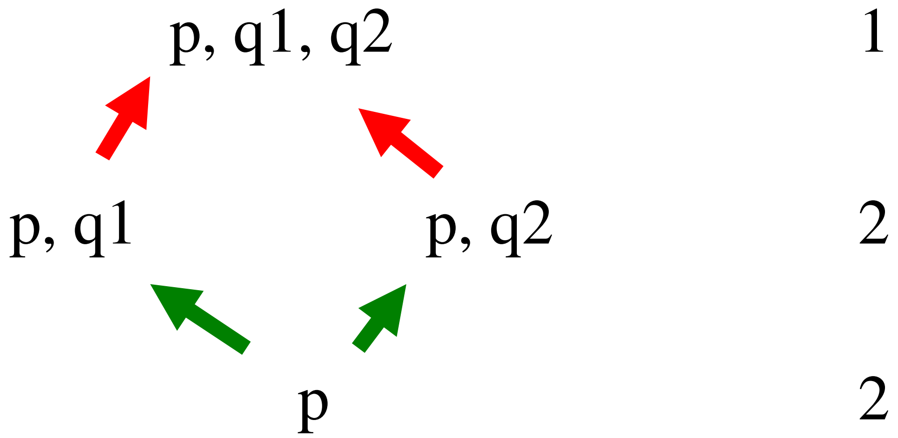


Possible even when q2 is equivalent to the negation of q1

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7. $\varphi \mid\!\not\sim \psi$ *if and only if* $v(\varphi) > 0$ *and* $\frac{v(\varphi\wedge\psi)}{v(\varphi)} < \epsilon$.

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The model has a transparent relation to logical validity.

The model is compatible with nonmonotonic logic (KLM style).

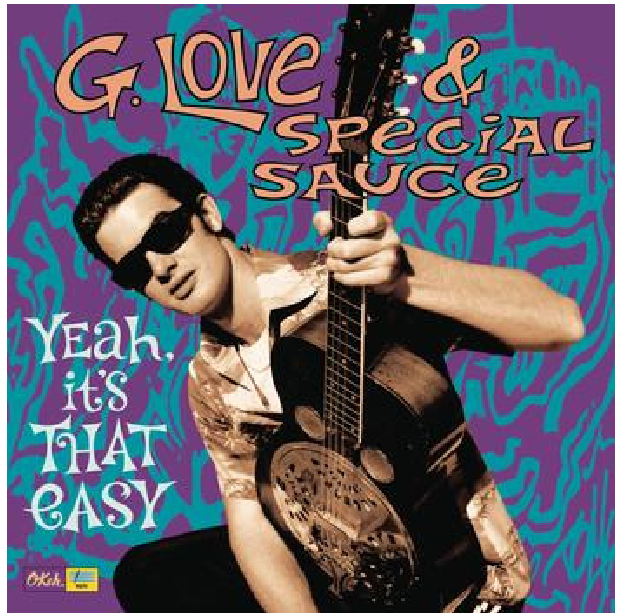
The model is compatible with standard probability theory.

Hey! Wait a minute!
Didn’t you say that
**computation of reasoning**
wasn’t really an issue?

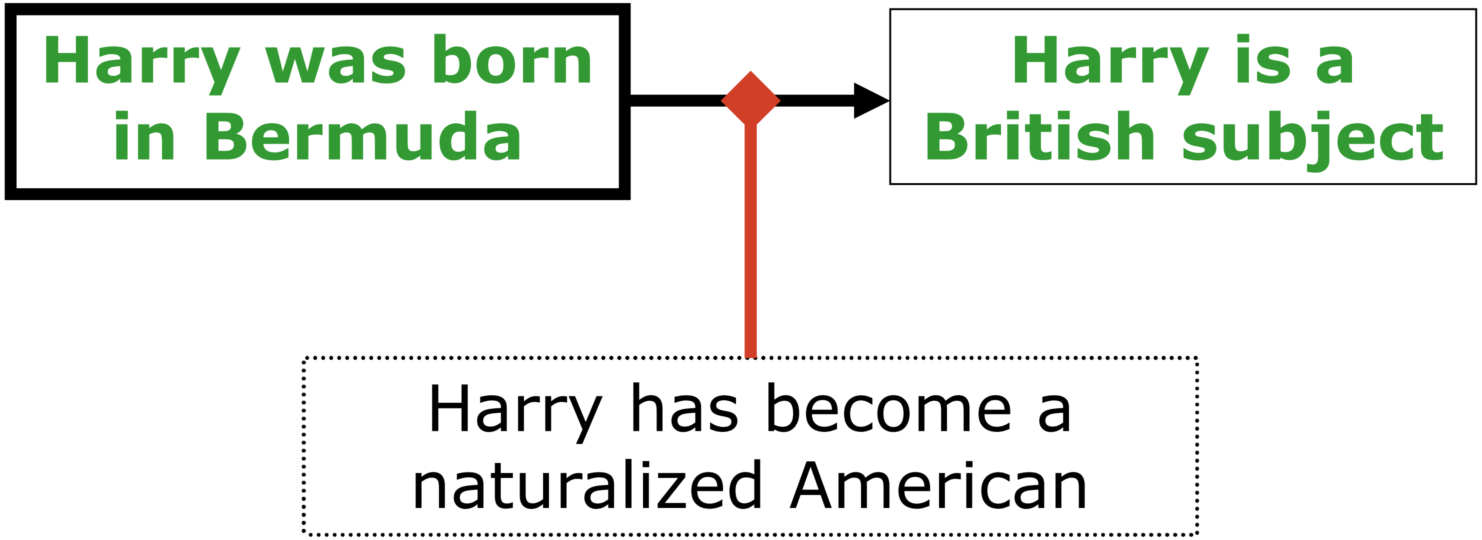
This all looks rather complex!!

Computation of reasoning is defeasible rule application.

1. Check the conditions.
2. Check the exceptions.
3. Draw the conclusion when the conditions apply and there is no exception.




Harry was born in Bermuda
Harry is a British subject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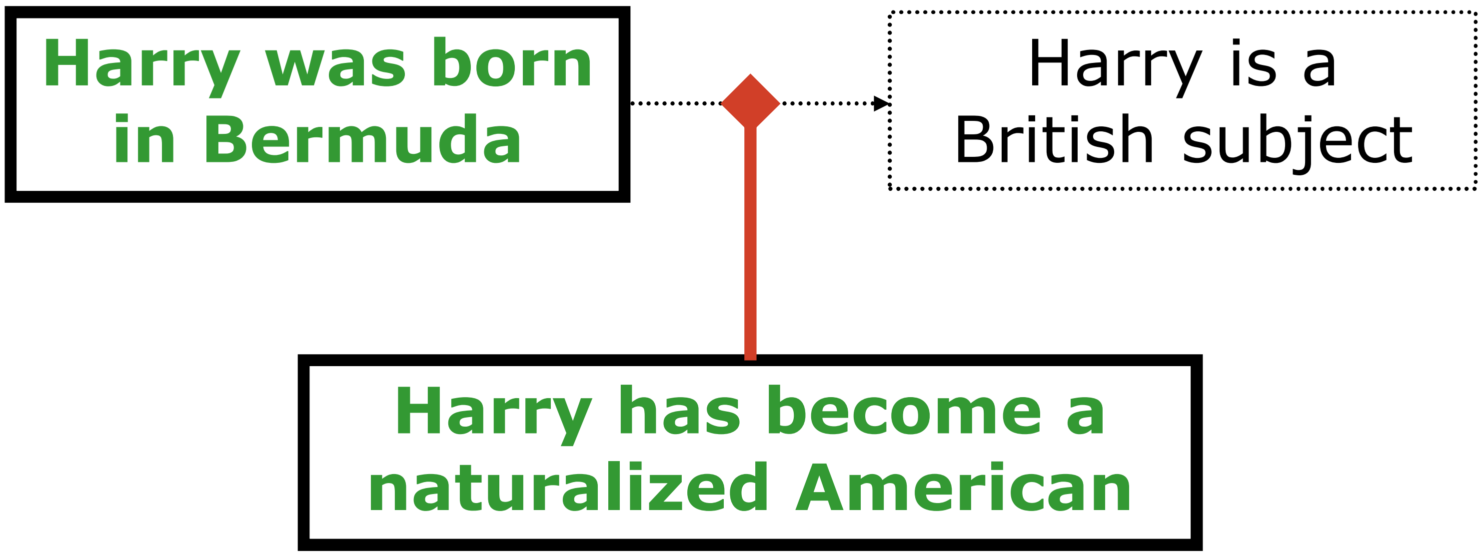
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The knowledge takes the form of rules and their exceptions.

Descriptive rules and exceptions can be found and tested as usual: by statistics.

Other rules and exceptions can be found by using reliable sources.

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It is possible to have one's cake and eat it too:

**Argumentation, logic, probability**

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